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# A matrix model for type 0 strings

**Jesús Puente Peñalba**

*Depto. de Física, Universidad de Oviedo*
*Avda. Calvo Sotelo 18, E-33007 Oviedo, Asturias, Spain*
*E-mail:* jesus@string1.ciencias.uniovi.es

Abstract: A matrix model for type 0 strings is proposed. It consists in making a non-supersymmetric orbifold projection in the Yang-Mills theory and identifying the infrared configurations of the system at infinite coupling with strings. The correct partition function is calculated. Also, the usual spectrum of branes is found. Both type A and B models are constructed. The model in a torus contains all the degrees of freedom and interpolates between the four string theories (IIA, IIB, 0A, 0B) and the M-theory as different limits are taken.

Keywords: Bosonic Strings, M-theory, String Duality, M(atrix) Theories.

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## Contents



## 1. Introduction

Matrix theory [7] has proven to be a useful tool to put together all the properties of the different known corners of M-theory. All the dualities arise very naturally and non-perturbative objects can be handled in a way that is more dynamical. All objects with different dimensionalities coexist in the same Yang-Mills action and therefore the "brane democracy" has a clear meaning.

Until recently, bosonic string theories were not part of the web of dualities that gathered all others as realizations of the same M-theory in different backgrounds. These theories were developed in refs. [4] and others. In this work, I shall concentrate in type 0A and type 0B bosonic string theories. The connection to the supersymmetric theories is found with an orbifold compactification of IIA [5] or M-theory [1] that describes these models in the appropriate limit of certain radius. There are also interpolating models constructed in [2] that relate the bosonic and the supersymmetric strings in a continuous way. However, there are still doubts on the validity of the interpolating model when the coupling is not zero and the present knowledge of the branes of the theory is not complete yet. There are, therefore, several reasons to try to make a matrix model of these strings and see how much information can be obtained.

One of the problems that can be solved with this matrix model is the behaviour when the coupling is not zero and, in particular, the way the tachyon acquires a mass and ceases to yield the theory inconsistent. Also, the duality relations are clearer because they explicitly appear in the lagrangean. The introduction of D-brane backgrounds is more problematic. They appear as objects of the theory but their interactions and, in particular, the couplings to the closed string fields still require a better knowledge of open type 0 strings.

## 2. Description of the orbifold

The idea is to try to make an orbifold of the Matrix Model of M-theory so that we obtain a non-perturbative, matrix model of type 0 strings. As argued in [1] these strings are compactifications of M-theory in an orbifolded circle. The first step is, therefore, to compactify the model in a circle through the construction of W. Taylor [8] and arrive at the hamiltonian

$$H = \frac{1}{2}\frac{R_+}{2\pi}\int_0^{2\pi} d\sigma_9 \, \mathrm{tr}\Big\{ p_i^2 + (DX_i)^2 + \theta^T D\theta + \\ + \frac{1}{R_9^3}\left(E^2 + \left[X^i, X^j\right]^2\right) + \frac{1}{R_9^{3/2}}\theta^T \gamma_i \left[X^i, \theta\right]\Big\}, \tag{2.1}$$

where everything is measured in units of $l_p^3 R_9^{-1}$ [9]. The question is now how to make a world-line orbifold of the Yang-Mills theory such that its effects are the same as the target orbifold of ref. [1]. As an ansatz, we shall work with a similar one and test its properties (spectrum, partition function, ...) to see if it, indeed, corresponds to a matrix realization of type 0 strings. The ansatz is to project out all bosons with odd world-line momentum. This way, world-line fermions effectively live in a circle of double size that of the bosons. Another way of looking at it is to consider that both live in the same circle but with different boundary conditions: bosons are periodic while fermions can be both periodic and anti-periodic. The conditions are similar to those one imposes around the compact Euclidean temporal direction when calculating at finite temperature. The difference is that fermions can also be periodic.

It is clear that these conditions break supersymmetry because they treat fermions and bosons in a different manner. One important consequence is that the zero modes of the oscillators do not cancel. Moreover, because of the fact that there are twice as many fermionic as bosonic modes, the vacuum has an infinite negative energy signalling the existence of a tachyon in the target theory.

In order to properly identify the string states, it is necessary to use other parameters

$$\alpha' = \frac{l_p^3}{R_9}, \qquad g_s = \frac{R_9}{\sqrt{\alpha'}}. \tag{2.2}$$

Now we can write the matrix string hamiltonian:

$$H = \frac{1}{2}\frac{R_+}{2\pi}\int_0^{2\pi} d\sigma_9 \,\mathrm{tr}\left\{p_i^2 + (DX_i)^2 + \theta^T D\theta + \right. \\ \left. + \frac{1}{g_s^2}\left(E^2 - \left[X^i, X^j\right]^2\right) + \frac{1}{g_s}\theta^T\gamma_i\left[X^i,\theta\right]\right\}. \qquad (2.3)$$

The perturbative string limit is the strong coupling limit of the Yang-Mills theory. To analyse the spectrum we have to go to the infrared limit. There, the scalars take vacuum expectation values and break the gauge symmetry to its Cartan subalgebra ($\mathrm{U}(1)^N$). This gives masses to all the fields so that in the extreme infrared, everything can be considered decoupled but the photons and the scalars that correspond to the commuting part of the matrices. This part will give us the free strings. There are eight scalars related by the SO(8) R-symmetry, one gauge field and their fermionic partners.

The spectrum of these fields can be obtained expanding them in a Fourier series. The hamiltonian is then a sum of fermionic and bosonic oscillators like

$$\frac{4\pi}{R_+}H = \sum_n \Pi_n^2 + X_{-n}n^2 X_n + \text{fermions}. \qquad (2.4)$$

It is not immediate to see the different sectors (NS-NS, R-R, ...) this way because the action we are dealing with is the equivalent of the Green-Schwarz action, not the Neveu-Schwarz-Ramond version. In the latter formalism, one can classify the fields according to the boundary conditions of the fermionic operator that is used to create them from the vacuum. With the G-S action, one just generates the whole spectrum acting with the supercharges over the vacuum state. This is impossible in this case because we do not have any. What we shall do is to compute the whole spectrum and see how it is organised in terms of the characters of the spin group.

## 3. The partition function

The best way of obtaining the spectrum is to calculate the internal partition function of the theory. Usual light-cone type 0 string theory is supposed to emerge in the large-$N$ limit of the gauge theory. In the limit of infinite coupling, the hamiltonian is truncated to the free part. Besides, we can identify

$$\frac{N}{R_+} = p_+\,. \qquad (3.1)$$

Apart from the $N$ diagonal fields of each class, there can also exist excitations with twisted boundary conditions like these

$$X(2\pi) = \mathcal{U}^\dagger X(0)\mathcal{U}\,, \qquad (3.2)$$

where $\mathcal{U}$ is an element of the $Z^N$ group of permutations. This group is the centre of the U($N$) gauge group and it is not broken even when the scalars take vacuum expectation values. For each element of the group there is a partition of the whole matrix in twisted states that represent single string states whose light-cone momentum is $p_+ = N_i/R_+$. The presence of states with several strings is a consequence of the fact that matrix models automatically include the second quantisation of the objects they describe. The partition function of the usual (non-matrix) strings calculated computing the path integral of the world-sheet fields is, in fact, a single string partition function. As we now want to compare the matrix results with the usual ones, we have to restrict the computation to single string states. With that, the final action is the Green-Schwarz light-cone action modified so that the fermionic fields can also be anti-periodic when going around the world-sheet coordinate.

We are now ready to find out the contribution of each field. It is easier if we imagine that all fields live in the bosonic circle (radius $R_9$) and let bosons be periodic and fermions both periodic and anti-periodic. The partition function can be seen as a trace over states that can be performed calculating the propagator of the fields with periodic (or anti-periodic) boundary conditions. The part of each boson is

$$Z_B = \frac{1}{\sqrt{\tau_2}} \frac{1}{\eta(\tau)\bar{\eta}(\tau)} \, . \tag{3.3}$$

This is obtained with the usual techniques of determinant regularization and Schwinger integral representations. It is quite standard, so I shall not explain every step. The fermions can be organised in spin structures according to the boundary conditions one chooses in both directions. One has to calculate the determinant of the Dirac operator on the torus (of complex parameter $\tau$) with eigenvalues $(n\tau + m)$ with $n$ an $m$ integers if the fields are periodic and integers plus one half if they are anti-periodic. After a proper regularization the results are

$$\begin{aligned}
Z_{\rm pp} &= \frac{\vartheta_1(\tau)}{\eta(\tau)} \quad \text{if they are periodic-periodic}, \\
Z_{\rm pa} &= \frac{\vartheta_2(\tau)}{\eta(\tau)} \quad \text{if they are periodic-antiperiodic}, \\
Z_{\rm ap} &= \frac{\vartheta_3(\tau)}{\eta(\tau)} \quad \text{if they are antiperiodic-periodic}, \\
Z_{\rm aa} &= \frac{\vartheta_4(\tau)}{\eta(\tau)} \quad \text{if they are antiperiodic-antiperiodic}
\end{aligned} \tag{3.4}$$

for the right moving part of each field. The first condition refers to the $\tau$ direction. After multiplying the bosonic and the fermionic left- and right-moving parts and summing over all the possible spin structures, the result is

$$Z = \frac{1}{2\tau_2^4} \left|\eta(\tau)\right|^{-24} \left( \left|\vartheta_2(\tau)\right|^8 + \left|\vartheta_3(\tau)\right|^8 + \left|\vartheta_4(\tau)\right|^8 \right) . \tag{3.5}$$

This is exactly the partition function of type 0 string theories. In our case the chirality of fermions is that of an A theory so this should be a description of type 0A string theory. With this function we can analyse the sectors of the spectrum reordering the Jacobi functions. One can put it this way

$$Z = \frac{1}{4\tau_2^4} |\eta(\tau)|^{-16} \left[ \frac{|\vartheta_2(\tau)|^8}{|\eta(\tau)|^8} + \frac{|\vartheta_2(\tau)|^8}{|\eta(\tau)|^8} + \left| \frac{\vartheta_3^4(\tau)}{|\eta(\tau)|^4} - \frac{\vartheta_4^4(\tau)}{|\eta(\tau)|^4} \right|^2 + \right.$$
$$\left. + \left| \frac{\vartheta_3^4(\tau)}{|\eta(\tau)|^4} + \frac{\vartheta_4^4(\tau)}{|\eta(\tau)|^4} \right|^2 \right] \tag{3.6}$$

which is

$$Z = Z_b \Big( \bar{\chi_S} \chi_C + \bar{\chi_C} \chi_S + \bar{\chi_V} \chi_V + \bar{\chi_I} \chi_I \Big) , \tag{3.7}$$

where the $\chi$ functions are the characters of the different representations of the target Lorentz group. Notice that all of them are bosonic. The characters tell us the rotational properties of each field. The $\bar{V}V$ sector is the gravitational one, it corresponds to the NS-NS sector of the supersymmetric theories and contains, among other fields, the graviton. The $\bar{I}I$ sector contains the tachyon and the two others are the two R-R sectors. We can now find more explicitly the mass spectrum and the degeneracies expanding the partition function in a series of exponentials of $\tau_2$, once the integral over $\tau_1$ is done. The first orders are

$$Z = \tau_2^{-4} \Big( q^{-2} + 192 + 1296 q^2 + \mathcal{O}(q^4) \Big) , \tag{3.8}$$

where $q = e^{-2\pi\tau_2}$. The first addend is divergent in the infrared (when $\tau_2$ is large) and is the contribution of the tachyon. The second addend corresponds to the massless fields (64 degrees of freedom come from each R-R sector and 64 more from the gravitational one). The partition function is a modular form, invariant, in particular under the inversion of $\tau_2$. This means that its ultraviolet behaviour is exactly the same as the infrared one. The infrared divergence of the tachyon is the same as the ultraviolet Hagedorn growth of the degeneracies. If the latter requires a small temperature to be regularized, the former would require a high one, so that finally, the theory is not sensible at any temperature; at least with $g = 0$.

## 4. The long-range potential

One of the effects of the appearance of a tachyon is that it is impossible to calculate a potential because the interchange of tachyons is extremely divergent in the infrared. Let us see that this is indeed our case. At one loop, it is not difficult to obtain the vacuum energy of the free fields in the compactified theory. The easiest way is to



JHEP08(1999)005

use the background gauge and count the number of fermionic and bosonic fields at each mass level. In the Schwinger representation the result is

$$V(r) = \frac{v}{4}\sqrt{\frac{R_+}{g_s}} \sum_n \int ds s^{-1/2} e^{-sr^2 R_+/g_s} \frac{1}{\sinh sv} \times \\ \times \left[ 16 e^{-s(2\pi n)^2 g_s R_+} \cosh sv - e^{-s(\pi n)^2 g_s R_+} \left(4\cosh 2sv - 12\right)\right] \tag{4.1}$$

when the coupling is small, one has to make a Poisson resummation and get

$$V(r) = \frac{v}{4} \frac{R_+}{\sqrt{\pi} g_s} \sum_n \int ds s^{-1} e^{-sr^2 R_+/g_s} \frac{1}{\sinh sv} \times \\ \times \left[ 8 e^{-sn^2/(4 g_s R_+)} \cosh sv - e^{-sn^2/g_s R_+} \left(4\cosh 2sv - 12\right)\right] \tag{4.2}$$

which in the infrared behaves as

$$-\frac{2}{\sqrt{\pi} g_s} \int \frac{ds}{s^2} e^{-sr^2 R_+/g_s} \,. \tag{4.3}$$

This is divergent and denotes the existence of the tachyon in the spectrum. It is possible for us to do perturbative computations in spite of dealing with a strongly-coupled theory because, in fact, we have deduced the form of the effective theory at low energies and it has turned out to be free. It is this free theory the one we have calculated the vacuum energy for and not the whole one. An analogous computation to this one was done in ref. [6]. There, the potential between two D3-branes was calculated. It was also divergent in a similar way and was given the same interpretation as a tachyonic effect. In order to give a sensible result, the authors of that reference introduced an infrared (in the image of the closed strings) cut-off and regularized the potential.

## 5. 0B strings and the dualities

We shall now be interested in describing models for 0B strings and how they can interpolate between them and the supersymmetric theories. For that, it is necessary to compactify the theory further in a second circle (third, taking into account also the light-like one) of radius $R_8$. After using again the Yang-Mills duality described in [8] we have a theory in $2+1$ dimensions. Around the second circle the fields have the usual periodic boundary conditions. This model interpolates between all five theories (IIA, IIB, 0A, 0B and M). If one chooses both radii to be small, the resulting infrared limit describes a type B theory. Which one depends on the particular choice of the string coupling constant

$$g_s^{0\mathrm{B}} = \frac{R_9}{R_8} \tag{5.1}$$

or

$$g_s^{\mathrm{IIB}} = \frac{R_8}{R_9} \,. \tag{5.2}$$

In this latter case it is the coupling constant of type IIB theory compactified in the orbifolded circle. When we take that circle to infinity, it will coincide with the supersymmetric case. The hamiltonian is

$$\begin{aligned} rH = \frac{1}{2}\frac{R_+}{2\pi} \oint d^2\sigma \,\mathrm{tr}\, \frac{1}{R_9 R_8} \Big[ & p_i^2 + (\partial_0 A_8)^2 + (\partial_0 A_9)^2 + R_9^2 (\partial_9 X_i)^2 + \\ & + R_8^2 (\partial_8 X_i)^2 + (R_9 \partial_9 A_8 - R_8 \partial_8 A_9)^2 + \\ & + \theta^T (R_9 \partial_9 + R_8 \partial_8) \theta + \text{interactions} \Big] \end{aligned} \tag{5.3}$$

with the constraint that bosons are periodic around both directions but fermions are periodic around $\sigma_8$ and can be periodic or anti-periodic around $\sigma_9$. This difference between the two directions spoils the $\mathrm{SL}(2,\mathbb{Z})$ symmetry of IIB strings. The self-duality is reduced to an S-duality between two different theories.

If we choose $R_9$ to be the quantum circle. We have to change variables this way

$$\begin{aligned} H = \frac{1}{2}\frac{R_+}{2\pi} \int_0^{2\pi} d\sigma_9 \int_0^{2\pi/R_8} d\sigma_8 \,\mathrm{tr}\, \frac{1}{R_8} \Big[ & p_i^2 + (\partial_0 A_8)^2 + (\partial_0 A_9)^2 + R_8^2 (\partial_8 X_i)^2 + \\ & + (\partial_9 X_i)^2 + (R_8 \partial_8 A_9 - \partial_9 A_8)^2 + \\ & + \theta^T (R_8 \partial_8 + \partial_9) \theta + \text{interactions} \Big] \,. \end{aligned} \tag{5.4}$$

As in the previous case, the limit where the strings give a good description is $g_s^{0\mathrm{B}} \to 0$, which is strongly coupled in the Yang-Mills. The extreme infrared limit is again described by the commuting fields and the structure of twisted states is also the same as that in the IIA (or 0A) theories (see [11, 10] where this aspect, and others, are studied for type IIA and IIB matrix string theories).

We can now analyse the spectrum. That of the scalars is

$$\begin{aligned} E^2_{\text{scalars}} &= \sum_{i=1}^{7} \frac{1}{2p_+} N^i_{n,m} \frac{1}{R_9} \sqrt{n^2 R_9^2 + m^2 R^8} = \sum_{i=1}^{7} \frac{1}{2p_+} N^i_{n,m} \sqrt{n^2 + m^2 \frac{R_8^2}{R_9^2}} = \\ &= \sum_{i=1}^{7} \frac{1}{2p_+} N^i_{n,m} \sqrt{n^2 + \frac{m^2}{(g_s^{0\mathrm{B}})^2}} \end{aligned} \tag{5.5}$$

with both $n$ and $m$ integers. We can see the IIB-like structure of $(p,q)$-strings. In fact there is no difference in the bosonic sector. The gauge field has the same spectrum as the scalars, but one has to consider also the purely topological sectors. There are three different elements of $F^{\mu\nu}$. The two electric components have configurations like

$$\frac{1}{(2\pi)^2} \oint d^2\sigma \partial_0 A_8 = \frac{k}{R_8}, \qquad \frac{1}{(2\pi)^2} \oint d^2\sigma \partial_0 A_9 = \frac{l}{R_9} \,. \tag{5.6}$$

With this we have to add

$$E^2_{\text{winding}} = \frac{1}{2p_+}\left(\frac{k^2}{R_8^2} + \frac{l^2}{R_9^2}\right) = \frac{1}{2p_+}\tilde{R}_8^2\left(k^2 + \frac{l^2}{(g_s^{0\mathrm{B}})^2}\right) \tag{5.7}$$

to the spectrum. One has to impose the condition of transversality of the electric field and the momentum

$$\vec{p}\vec{E} = 0 \tag{5.8}$$

which means that

$$a(k,l) = b(-n,m)\,, \tag{5.9}$$

where all numbers are integers. As a result, $b$ is the winding number, $a$ is the excitation number and the true tension is $a^{-1}(n,m) = b^{-1}(k,l) = (p,q)$. A similar quantisation condition holds for the magnetic field. In this case the easiest way to obtain the energies is to follow [13] and to quantise the momentum of the string along the eighth direction. Adding up every term, we have obtained

$$\begin{aligned} E^2 &= \frac{1}{2p_+}\left\{\sum_{i=1}^{8}\left[N_{n_i}^{(p,q)} n_i^{(p,q)}\sqrt{p^2 + \frac{q^2}{(g_s^{0\mathrm{B}})^2}} + \omega^2 \tilde{R}_8^2\left(p^2 + \frac{q^2}{(g_s^{0\mathrm{B}})^2}\right) + (p^i)^2\right] + \frac{k^2}{\tilde{R}_8^2}\right\} = \\ &= \frac{1}{2p_+}\left[\sum_{i=1}^{8}\left(N_{n_i}^{(p,q)} n_i^{(p,q)} 2\pi T_{(p,q)} + (2\pi\omega\tilde{R}_8)^2 T^2_{(p,q)} + (p^i)^2\right) + \frac{k^2}{\tilde{R}_8^2}\right], \end{aligned} \tag{5.10}$$

where

$$T_{(p,q)} = \frac{1}{2\pi\alpha'}\sqrt{p^2 + \frac{q^2}{(g_s^{0\mathrm{B}})^2}} \tag{5.11}$$

this is exactly the bosonic spectrum of type B theories compactified in a circle. The fermions make the difference because their "$p$" tensions can also be half integers. This is what breaks the $\mathrm{SL}(2,\mathbb{Z})$ symmetry of type IIB, but not completely because there are still bound states of 0B an IIB strings. When one takes the coupling to be very small, one has to keep only the $(1/2,0)$-strings. The spectrum of the $(p,q)$-strings depends on whether $p$ is integer or not. If it is, fermions are periodic around the world-sheet and the spectrum is that of the IIB strings while if $p$ is half an odd number, the multiplets are those of type 0B strings.

This also shows that, as proposed in [1], the tachyon acquires a mass when the coupling is not small. The reason is that when $g_s^{0\mathrm{B}}$ is large, one should make a flip (an S-duality) and consider $R_8$ to be the quantum circle. The hamiltonian is the same again but the fermions are periodic in the world-sheet, and therefore generate the IIB supersymmetric spectrum. The first level is the massless one so we conclude that, at least at large coupling, the theory is sensible and does not contain tachyons. The swap of the role of the two circles depends on their sizes and determines the phase structure of M-theory in this compactification (see figure 1).

When one takes the coupling constant to zero and keeps only the $(1/2, 0)$-strings, one can calculate their partition function. As in the supersymmetric case [11], it is necessary to integrate over the dual of the photon field after adding a topological contribution

$$\epsilon^{\mu\nu\lambda} F_{\mu\nu} \partial_\lambda \phi^8 \, . \tag{5.12}$$

With this new scalar $\phi$, the SO(8) symmetry is restored. Also, the fermions change their chirality as is usual in a T-duality, which is, ultimately, what we are doing. With this, it is immediate to see that the partition function of these strings is exactly the same as that of the 0A computed above. The only difference is the chirality of fermions. As a result, this model reproduces type 0B string theory.

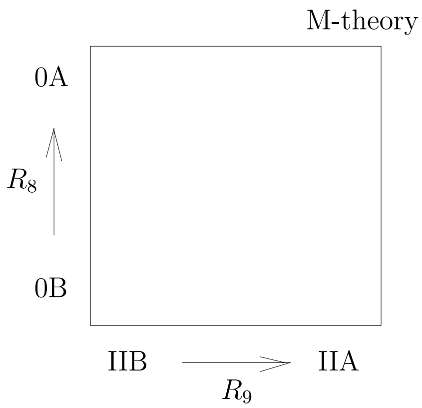


**Figure 1:** Square of theories related by dualities in the orbifolded torus.

The type IIB limit is the opposite ($g_s^{\mathrm{IIB}} = R_8/R_9 \to 0$). The type IIA string can be achieved sending now $R_8$ to infinity. With a dimensional reduction of the $\sigma_8$ direction, the hamiltonian simply reduces to that of ref. [9]. Figure 1 sketches all the dualities on this torus.

## 6. Non-perturbative states

In usual supersymmetric matrix string theories one can study non-perturbative objects finding out the fluxes of fields that contribute to the central charges of the supersymmetry algebra. In this case we do not have that guide. However, similar fluxes of fields can be seen to act as Dirichlet objects and therefore they are unstable non-perturbative effects with renormalised masses and charges. In fact, in the derivation of their existence and their behaviour as walls where the open strings can end [9, 12, 13] only the bosonic sector is needed, so one can extrapolate known results to this case.

We have already seen one of the non-perturbative objects, the D-string. Among others, the usual IIB massless spectrum has appeared. From the point of view of the 0B corner ($g_s = R_9/R_8$), they have acquired a large mass due to the fact that they are strongly coupled, but still keep their form as a supersymmetric multiplet, including the fermions.

Another interesting case is that of D0-branes in 0A theory. The flux related to them, as well as to their bound states is

$$\frac{1}{2\pi} \oint d\sigma E_9 = n \, , \tag{6.1}$$

where $E_9$ is the electric field and $n$ is the charge of the bound state. As argued in [12], the presence of a constant electric field forbids the other fields from carrying momentum so that there cannot be any excited oscillators in the Yang-Mills and the object with that flux has to be point-like. What makes it act as a Dirichlet point is precisely that fact: when a twisted state is formed between a perturbative string and a D0-brane (that is, between two matrices with and without constant background electric fields), there cannot be a flux of momentum (oscillator number) across the part of the world-sheet with constant electric field (D-particle) so that it acts as a wall against which the momentum of the string rebounds; that is why it provides it with Dirichlet boundary conditions.

Membranes in 0A an D3-branes in 0B theories are related to non-commutative configurations of fields like

$$\Phi = \frac{i}{4\pi} \int d\sigma_8 \, \mathrm{tr} \left\langle \left[ X^7, X^6 \right] \right\rangle = T_2 A \,. \tag{6.2}$$

In the 0B case one also has to integrate over the $\sigma_9$ direction, which is the third world-volume dimension of the D3-brane. The other branes are related to other fluxes exactly like in ref. [9, 13].

## 7. Interactions

As in ref. [9, 10], interactions appear when the condition of infinite coupling in the Yang-Mills theory is relaxed a bit. With that assumption there are special points in the moduli space where part of the broken gauge symmetry is restored. This happens whenever at least two eigenvalues of the matrices coincide for one value of $\sigma_9$. The symmetry is enhanced to SU(2) and asymptotically the effect is the interchange of the eigenvalues, that is, the action of an element of the unbroken centre of the group, $Z_N$. This means that also in this case the interactions are local in the world-sheet and appear in the target space as splitting and joining processes. In the absence of any supersymmetry it is hard to find the operators in the CFT that act as vertices. That would involve dealing with a non-perturbative expansion of a Yang-Mills theory without any supersymmetry. Also, some relation should be found between the running of the coupling constant in the Yang-Mills and the string theory. The only thing we can say, for the moment, is that, in the infrared, they can be given a geometrical interpretation as string interactions.

## 8. Conclusions

The model presented here is a simple orbifold construction of the matrix model of strings that describes closed type 0 strings as infrared fixed points of one and two dimensional Yang-Mills theories. The world-sheet action is not supersymmetric but it contains fermions that, together with bosons generate the correct spectrum. The partition function is modular invariant.

The effect of the tachyon is to provoke an infrared divergence both in the partition function and in the long-range gravity potential.

The two-dimensional Yang-Mills theory describes the IIB/0B dual system with $(p, q)$ bound states that can be purely bosonic or fill up a supersymmetric multiplet. In particular IIB or 0B strings appear as non-perturbative excitations of the dual theory. As a conclusion, the 0B theory, although completely bosonic perturbatively, does contain fermions when the coupling is not zero. Also, this model contains, through dimensional reductions, the two other corners of the duality, the type A theories.

Other branes can also be described as fluxes. Without supersymmetry, these objects are not protected against decay or renormalization of their masses and charges, nevertheless they are true excitations of the model. A more profound study would need a better knowledge of open type 0 strings.


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## References


[1] O. Bergman and M.R. Gaberdiel, *Dualities of type 0 strings*, *J. High Energy Phys.* **07** (1999) 022 [hep-th/9906055].

[2] J.D. Blum and K.R. Dienes, *Strong/weak coupling duality relations for non-supersymmetric string theories*, *Nucl. Phys.* **B 516** (1998) 83 [hep-th/9707160].

[3] Y. Imamura, *Branes in type 0/type II duality*, hep-th/9906090.

[4] L.J. Dixon and J.A. Harvey, *String theories in ten dimensions without space-time supersymmetry*, *Nucl. Phys.* **B 274** (1986) 93;
O. Bergman and M.R. Gaberdiel, *A non-supersymmetric open string theory and S duality*, *Nucl. Phys.* **B 499** (1997) 183 [hep-th/9701137];
L. Alvarez-Gaume, P. Ginsparg, G. Moore and C. Vafa, *An* $O(16) \times O(16)$ *Heterotic String*, *Phys. Lett.* **B 171** (1986) 155;
N. Seiberg, E. Witten, *Spin structures in string theory*, *Nucl. Phys.* **B 276** (1986) 272.

[5] R. Rohm, *Spontaneous supersymmetry breaking in supersymmetric string theories*, *Nucl. Phys.* **B 237** (1984) 553.

[6] A.A. Tseytlin and K., *Effective potential in non-supersymmetric* $\mathrm{SU}(N) \times \mathrm{SU}(N)$ *gauge theory and interactions of type 0 D3-branes*, *Phys. Lett.* **B 457** (1999) 77 [hep-th/9902095].

[7] T. Banks, W. Fischler, S.H. Shenker and L. Susskind, *M-theory as a matrix model: a conjecture*, *Phys. Rev.* **D 55** (1997) 5112 [hep-th/9610043].

[8] W.I. Taylor, *D-brane field theory on compact spaces*, *Phys. Lett.* **B 394** (1997) 283 [hep-th/9611042].

[9] R. Dijkgraaf, E. Verlinde and H. Verlinde, *Matrix string theory*, *Nucl. Phys.* **B 500** (1997) 43 [hep-th/9703030].

[10] L. Motl, *Proposals on non-perturbative superstring interactions*, hep-th/9701025.

[11] T. Banks and N. Seiberg, *Strings from matrices*, *Nucl. Phys.* **B 497** (1997) 41 [hep-th/9702187].

[12] J. Puente Peñalba, *Non-perturbative thermodynamics in matrix string theory*, hep-th/9904094.

[13] T. Banks, N. Seiberg and S. Shenker, *Branes from matrices*, *Nucl. Phys.* **B 490** (1997) 91 [hep-th/9612157].